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ABSTRACT

The focus of this study was on the processes students used in making mathematical conjectures for five divergent problem situations. The relationship of the processes to creativity factors across the problems was also examined. The results of various cluster analyses and correlational measures indicated that students use different sequences of processes for different problems; some problem types elicit more pattern searches than others; and when the processes were viewed in ordered pairs and triples, various sequences resulted. The creativity measures were all significantly correlated, but there were differences on the relationships between the creativity factors and various processes used. (Author/MP)

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PROCESSES INVOLVED IN MATHEMATICAL DIVERGENT PROBLEM SOLVING

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General methods and techniques for solving problems have been discussed by mathematics educators since the time of Socrates. In many of the interchanges, problem solving has been related to one or more of the factors of creativity (Boychuck,1974; Kilpatrick, 1969; Torrance, 1962). Other studies have focused on the heuristics used by students engaged in problem solving activities (Boychuck, 1974; Kilpatrick, 1968; Webb, 1975) and whether or not these and related heuristics are teachable (Kantowski, 1977; Lucas, 1974).

The present study was an extension of Boychuck's work and involved the relationship between conjecturing behaviors and the creativity factors of fluency, flexibility, originality, and organization. The study also built on the work of Kilpatrick and Webb in the description of the cognitive processes employed by students involved in problem solving situations.

The Study

The Objectives

The study had two general objectives: (a) to examine both the processes and the sequences of processes students used while making conjectures for five different open-ended mathematical situations and (b) to search for relationships in the clustering patterns resulting from the students' creativity scores and sequences of cognitive processes.

The Subjects

The subjects used in the study were 30 ninth-grade mathematics students enrolled in three different high schools. The students were enrolled in three different high schools. The students were selected by their classroom teachers on the basis of their meeting the criteria of high mathematics grades, above-average intelligence, and active participation

in classroom activities.

The Procedures

A pilot study was conducted using 10 ninth-grade mathematics students as subjects. The purposes of the pilot study were to: (a) choose the problems to use in the study, (b) develop a technique for recording the students' manipulative behaviors when working with concrete objects, (c) improve the directions given to the students, (d) develop the process scoring system, and (e) develop scoring procedures for creativity.

In the next phase of the study, each of the 30 students was interviewed separately for approximately one hour. During this period of time, the student was presented with five open-ended mathematical situations and was asked to think aloud while making conjectures. Verbal comments were audio recorded. The tapes were then transcribed and nonverbal behaviors were interwoven with verbal statements. The problem solving processes were coded and the conjectures were scored for the creativity factors.

The Problems

The five problems used in the study were selected on the basis of their ability to: (a) elicit student responses, (b) involve a wide range of mathematical concepts, facts, and skills, (c) be new to, but appropriate for, the students, (d) provide an opportunity to observe students discovering relationships, comparing and contrasting facets of the problems, drawing conclusions, or stating generalizations. The problems selected were chosen from a group of nine problems used in the pilot study. The five problems were a Measurement Situation (ME), Pascal's Triangle (PC), Tessellations (TS), Number Systems (NS), and Polyhedra (PO). See Figure 1 for one of the problems given to the study participants.

Insert Figure 1 Here 4

The Problem-Process Sequence Codes

Since the present study differed from both the Kilpatrick (1968) and Lucus (1974) studies, their process sequence codes were not directly applicable. A modification of Lucas' coding system was used. This coding system for identifying the verbal and nonverbal aspects of the student's cognitive behaviors had 40 categories--25 for verbal behaviors, 10 for nonverbal behaviors, and 5 to indicate transitions between behaviors.

These 40 categories were further grouped into six broad classes: Analysis/Synthesis, Interpretation, Logical, Terminal, Operational, and Procedural. These six classes were developed to assist in the analysis of the sequences of processes used by the students.

The 40 behavioral categories, grouped by the classes, are given in Table 1. An underline symbol indicates that the process was a nonverbal one.

Insert Table 1 Here

Inter-rater reliability coefficients between the two experimenters' coding were calculated by dividing the number of processes agreed upon by the average number of processes coded. The resulting quotient across problems was 0.75, with the range of the quotients across problems being 0.71 to 0.84.

The Creativity Scoring

The transcripts were scored for five creativity factors using a variety of methods of analyzing the student's verbal and nonverbal actions.

Fluency. A student received a point for each verbal mathematical statement made while working the probelm. Repeated statements, non-mathe-

matical comments, and incorrect remarks were not counted. The total number of fluency points for each student was calculated for each problem. The inter-rater reliability for this acoring process for the two experimenters was 0.94.

Originality. The students' responses that received points for fluency were then scored for originality on the basis of the number of students giving a particular statement. A mathematical statement given by only one student received a score of 4; a statement given by two students received a score of 3; and a statement given by three students received a score of 2. If four to six students had made a specific remark, it was given a score of 1. Remarks made by more than six students were not awarded any originality score. The total number of originality points awarded to each student was summed for each problem.

Flexibility. The scoring for flexibility was done following the scoring for originality. Each response that received a point for fluency was categorized with respect to its mathematical point of view of the problem. The flexibility score for a student was the number of different categories the student used. The inter-rater reliability for the two experimenters for flexibility was 0.90.

Organization. Organization was scored by awarding points for occasions on which the students posed a general or specific problem, tested possible combinations when working with descriptive guesses, or gave items in a trial-and-error pattern search and/or the generalization.

Total Mathematical Creativity. The total mathematical creativity score was the sum of the scores given for the creativity factors of fluency, flexibility, originality, and organization.

Hypotheses

The coding of the behaviors and scoring for the creativity factors were directed toward the testing of the follow hypotheses:

1. The sequence of processes will not be different for each problem.
2. The sequence of processes will not be different for each student.
3. The students will not use a different organized procedure for each problem.
4. Students' performances on organization, fluency, flexibility, originality, and total mathematical creativity will not be significantly correlated.
5. Students scoring high in
   (a) organization will not use different sequences of processes.
   (b) fluency will not use different sequences of processes.
   (c) flexibility will not use different sequences of processes.
   (d) originality will not use different sequences of processes.
6. there will not be a significant correlation between the number of processes employed by a student and the student's scores for fluency, flexibility, organization, originality, or total mathematical creativity.

Data Processing.

Following the interviews, the audio tapes were transcribed, nonverbal behaviors were interwoven with verbal statements, and the resulting transcripts were coded and scored with respect to the processes and creativity categories. The processes employed by each student were recorded in sequence.

These sequences were analyzed by searching for patterns of usage of: (a) the individual processes, (b) various ordered pairs of the processes,

and (c) various ordered triples of the processes. Each of these three methods of analysis of the processes was further partitioned to consider the number and percentage of each process, pair, or triple for each problem, for each student, and for each student by each problem. These analyses were carried out using the SPSS subprogram CROSSTABS (Nie, Hull, Jenkins, Steinbrenner, & Bent, 1975).

Various cluster analyses were carried out on the resulting data using the UCLA Biomedial Computer Program BMDP2M--Cluster Analysis on Cases (Dixon, 1975).

Results

Process Results

Single Processes. The analysis of the process usage across students and problems showed that the most frequently occuring process was description (Dc). This process occurred 14.6% of the time. The next eight highest ranking processes in order of their percentages of use were: finished sequence with completion (.), 10.7%; nonverbal comparison (C), 7.9%; nonverbal manipulation (Ma), 7.8%; nonverbal arranging (Ar), 6.6%; staring or pausing (-), 5.8%; qualification or explaining (Qu), 5.8%; finished sequence without completion (/), 5.4%, and trial and error search for a pattern (Ti), 5.0%.

These nine processes were the only processes to exceed 5% of the total process usage. The tenth process (Op, nonverbal mental calculation) was only used 3.1% of the time.

When the 40 processes were divided into the six classes given in Table 1, the percentage for each class by problem was as given in Table 2.

Insert Table 2 Here

Using the 20 processes that were employed most frequently by the students as variables, a cluster analysis was performed to see how the students clustered on the processes. Three clusters emerged from this analysis, accounting for 14 of the 30 students. The other 16 students were isolated in their patterns of usage.

The students in cluster 1 were high in Analysis/Synthesis and Procedures and low in Interpretation. (High and low refer to above and below the mean standard score for each process variable in that class.) Students in cluster 2 were high in Interpretation and low in Terminal processes. Students in cluster 3 were low in Analysis/Synthesis and Interpretation.

Sequences of Processes. The most frequently occurring ordered pairs of processes were examined next. This analysis was carried out by categorizing each component of the pairs and triples according to the six classes of processes.

The 87 pairs which occurred 20 or more times were analyzed by focusing on the first process and its class. Then for each class, the frequency of the class of the second process was determined. Whenever the first process was Analysis/Synthesis, or Operational, or Procedural, then it was the most frequently followed by an Analysis/Synthesis or Operational process. Whenever the first process was Interpretation or Logical, then the most frequent second process was Terminal. When the first process was Terminal then most frequent second process was Operational.

The pairs were then analyzed by focusing on the second process and its class. Then for each class, the frequency of the class of the preceding process was determined. Whenever the second process was Terminal or Procedural, then it was most frequently preceded by Analysis/Synthesis or Terminal processes. Whenever the second process was Analysis/Synthesis or Logical, then it was most frequently preceded by an Operational process. When the second process was Interpretation it was most

frequently preceded by an Analysis/Synthesis or a Logical process. When the second process was Operational, it was most frequently preceded by a Terminal process.

Following this, the triples which occurred ten times or more in the listing of triple usage were analyzed. This analysis focused on the middle process and its class. Then the frequencies of the classes of the preceding and succeeding processes were determined. When the middle process was Analysis/Synthesis, the most frequent preceding process was Operational and the most frequent succeeding process was Terminal. When the middle process was Interpretation, the most frequent preceding process was Analysis/Synthesis and succeeding was Terminal. When the middle process was Logical, the most frequent preceding process was Operational and succeeding was Interpretation or Logical. When the middle process was Terminal, the most frequent preceding process was Terminal and succeeding was Operational. When the middle process was Operational, the most frequent preceding process was Terminal or Operational and succeeding was Analysis/Synthesis or Operational. When the middle process was Procedural, the most frequent preceding process was Terminal and succeeding was Analysis/Synthesis.

The remaining study of the role of processes in sequences was limited to the 31 pairs and 24 ordered triples which occurred most frequently across all problems by all students. Hypothesis 1 was first examined by testing for differences in the percentage of usage of the 31 pairs and the 24 triples for students across problems. Using Kendall's coefficient of concordance (Siegel, 1956) the ordered pairs resulted in $W = 0.318$ ($p < 0.05$). The value for the analysis of the ordered triples was $W = 0.177$ ($p > 0.05$). This suggested that when the problems

are considered as a group, student process orderings by pairs are quite similar, but that the ordering for triples are different.

Hypothesis 1 was then analyzed for each pair of problems by finding the Spearman rank correlation coefficients for the usage of the 31 ordered pairs and the 24 triples by student by problem. These coefficients are given in Table 3.

Insert Table 3 Here

The sequence of the ordered pairs of processes appears to differ for the following pairs of problems: Measurement and Pascal's Triangle, Measurement and Tessellations, Number Systems and Tessellations, Pascal's Triangle and Polyhedra, and Pascal's Triangle and Tessellations. The sequence of usage of ordered triples by problem pairs across students was judged different for all pairs of problems except for Number Systems and Pascal's Triangle and for Polyhedra and Tessellations.

Hypothesis 2 was not examined statistically. A visual examination of the individual student's usage of the processes in sequence indicated differences. Further study of the 31 ordered pairs and 24 triples was done by performing two cluster analyses. The ordered pair analysis led to five clusters of students accounting for 14 of the 30 students. The analysis for the ordered triple usage led to six clusters accounting for 17 of the 30 students.

Creativity Scoring Results

Organization. The analysis of organization schemes by problems focused on four specific heuristics employed by the students: posing a general problem, posing a specific problem, descriptive guesses covering all possibilities, and trial-and-error searches leading to a generalization. The total number of points awarded for these four organizational procedures

was determined by student by problem.

A two-way analysis of variance was performed to test the hypothesis that organizational procedures would not vary across the problems. Table 4 contains the results of that analysis.

Insert Table 4 Here

Hypothesis 3 was rejected because the procedure F-value exceeds the critical value for the F-test at the 0.01 level of significance. Scheffé's Test for Multiple Comparisons (Edwards, 1968) was then applied to the problem sums to determine the nature of the procedures across problems. The results of Scheffé's analysis indicated significant differences ($p < 0.05$) in organized procedure usage on all pairs of problems except the pairs Pascal's Triangle and Number Systems and Pascal's Triangle and Tessellations.

Although Scheffé's test showed significant differences in the elicitation of organizational procedures by problems, the type of organizational scheme called forth by each problem was determined via the graphing process shown in Figure 2.

Insert Figure 2 Here

Hypothesis 4 posited that the creativity factors of fluency, flexibility, originality, organization, and total mathematical creativity would not be significantly correlated. This hypothesis was tested by finding the Pearson Product-Moment Correlation for each pair of factors. Table 5 contains the results of this analysis. Since all of the coefficients calculated were judged to be significant at the 0.001 level, the null hypothesis was rejected.

Insert Table 5 Here

A cluster analysis was performed using the five creativity factors as variables. Six clusters of students emerged, accounting for all but three of the 30 students. Figure 3 shows the relationship of the six clusters when compared on each of the five creativity categories. The cluster performance for each variable is the mean standard score of the cluster's performance in that category.

Insert Figure 3 Here

An analysis of the cluster graphs indicated that with one exception the clusters were linearly ordered in their performance on each of the five creativity categories. Clusters one, two, and three performed above the mean standard score on all five of the categories. Clusters five and six performed below the mean standard scores on the five categories. Cluster four was above the mean standard scores on fluency and flexibility, but below them on originality, organization, and total mathematical creativity.

Process and Creativity Scoring Results

Hypothesis 5 was evaluated by examining the students' performances for each of the creativity factors. For each factor the students scoring in excess of one standard deviation above the mean score for that factor were labeled as the high students. Then using the most frequently occurring 31 ordered pairs and 24 ordered triples, the percentage of occurrence of each pair and triple was determined for each creativity high group and for each group of remaining students.

The hypothesis was then tested by analyzing the ordering of the

usage of ordered pairs and ordered triples by the Spearman rank order correlation (Siegel, 1956). The results of this correlational analysis are shown in Table 6.

Insert Table 6 Here

The results of the analysis showed that the sequences used by the high students and the other students did not differ for the ordered pair sequences for any of the creativity factors. The analysis showed that different sequences of ordered triples were used by the high students and the other students for fluency, flexibility, and originality. In organization, as viewed through the triples, the sequences used by the high students were judged to be significantly like that used by the other students.

An analysis of the composition of the high groups showed that the high fluency and high originality groups were identical and both contained subjects 8, 11, 20, and 29. The high flexibility group contained subjects 8, 11, 13, 19, 20, 28, and 29. The high organization group contained subjects 11, 13, 20, 28, and 29.

Hypothesis 6 suggested that the number of processes employed by a student would be independent of the student's creativity scores. This hypothesis was examined by finding the Pearson Product-Moment correlation between the number of processes which were coded for each student and the student's scores for each of the creativity factors and for the total mathematical creativity score. Each of these comparisons was judged to be statistically significant. Thus Hypothesis 6 was rejected. Table 7 contains the results of this analysis reported by creativity score and problem type.

Insert Table 7 Here

Discussion

General Patterns of Processes

From the findings, it was evident that certain types of processes were used more than others and that different mathematical situations elicited different types of processes. The Pascal's Triangle and Number Systems problems elicited a great deal of staring behavior as compared to the other problems because the students did not have objects to manipulate. Completion of thought was fairly evenly distributed across problems so this particular behavior was not utilized more in one situation than in another. The problems involving concrete materials elicited Operational processes most because of the processes Ar (arranging), C (comparing), and Me (measuring) which could be used more when working with materials.

A cluster analysis using the most frequently employed 20 processes yielded three different groups of students. However, the fact that 16 of the 30 students could not be grouped at all showed that most of the students were individualistic.

When the most frequent ordered pairs of processes were studied, it was seen that certain types of processes were more commonly preceded or succeeded by certain types of processes. This would indicate non-random patterns of thinking and behaving in these students.

Sequences of Processes

When the sequence of the most frequent 31 ordered pairs was studied, it was found that the most used pairs for the Measurement Polyhedra, and Tessellations problems involved more of the manipulative processes, whereas the ones for the Pascal's Triangle problem involved more completion

of thought, staring, description, mental calculation, and trial-and-error. The pairs for the Number Systems problems involved more completion of thought, staring, description, mental comparisons, trial-and-error, and explaining.

For the most used ordered triples in the Measurement, Polyhedra, and Tessellations problems, the processes involved more of the manipulative processes, mental counting, and at random trial-and-error.

The cluster analyses again showed many isolated students and even of the groups that were formed, many of them were groups of two students. This seemed to indicate that even when similarities existed between students, the most common similarities were between pairs of students.

Different Problems Eliciting Different Heuristics

Both the Number Systems and Pascal's Triangle problems, as compared to the Measurement and Polyhedra problems, elicited more searching for patterns and making generalizations through a trial-and-error search. In general, this heuristic was used more across all problems.

Factors of Creativity

Since all pairs of creativity factors were significantly and positively correlated, it would seem that either these factors are not separate in mathematics or the methods used did not adequately distinguish between these factors.

The cluster analysis on creativity factors accounted for 27 of the 30 students. However, the six groups were clearly distinct from one another and almost the only difference between them was that the mean standard scores ranged from -1.5 to 2. This occurrence could be related to a student scoring high on one creativity factor and also scoring high on the other factors.

It is interesting to note that even among the high scoring mathematics

students, there was a diversity in creativity, However, even if some of the scores were low relative to the group, they might not be low in comparison to a cross section of all students. In Getzels and Jackson's study (1962) a relatively low relationship between high IQ and high creativity was claimed. When IQ's were examined however, it was seen that the high creativity group had a mean IQ of 127 which is still well above the average. A similar phenomenon could have occurred in this study.

High Scoring Creativity Students Compared to Others for Processes

The findings indicated that the high fluency (or originality, since both groups contained the same students) and the high flexibility groups were different from the other students in the sequence of ordered triple processes used. In comparing the top five triples, the high fluency (or originality) students used more mental computation, mental counting, and mental comparison combined with description. The other students used more completion of a thought followed by staring and manipulation. The high flexibility group used mental computation and mental comparison combined with description. They also used manipulation and staring combined with explaining followed by a completion of a thought. The other students used more completion of a thought and staring in combination with description, mental comparing, manipulation, or explaining.

The most often occurring triple for all students involved trial-and-error, which seemed to indicate a dominance of this process.

Commonalities Among Groups of Students

The findings indicated that some students who were low in creativity had common characteristics different from those students high in creativity. The low creatives had a low use of Analysis/Synthesis and Interpretation processes whereas the high creatives had a high use of Interpretation processes and a low use of Terminal processes. In comparing ordered triples

for the two groups, the low creatives tended to use generalizing followed by explaining before completion of a thought whereas the high creatives tended to use mental counting followed by description or description followed by explaining.

Implications

Since this study was exploratory in nature, it would be presumptuous to make any definite conclusions about the findings. Perhaps the most far-reaching implication is that studies such as this one need to be expanded and more of them done. Since this study examined only a subgroup of ninth-grade students, more research is needed on mathematical problem solving processes used by students at all ages and ability levels.

The fact that different problem situations evoked different processes and different heuristics has some implications for education. An awareness of different techniques may be helpful to a teacher in the structuring of tasks or in the presentation of different models for problem solving. The specific tasks used in this study could be used in the classroom to generate interest in a topic or for the teacher to learn about the students' mathematical background.

The area of creativity in mathematics also needs more research in relation to problem solving. Needed are improved methods of measuring mathematical creativity and especially originality if tests other than Torrance's are used.

The fact that some of the high creative students used some different processes compared to some of the other students implies some differences between these students and the other students. Possibly these students were at a different developmental level in relation to their problem solving abilities. However, they were not studied in enough depth to reach any conclusions.

An important aspect of research that this study did not examine is the problem solving process in mathematics as it would occur in a classroom. Interactions between students and between students and teachers need to be studied so that some picture of classroom problem solving behavior could be obtained. Ethnographic techniques (Magoon, 1977; Wilson, 1977) might be most useful in assessing the total picture as it relates to problem solving learning and instruction. Perhaps this type of study would have the greatest implications for education.

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Table 1

Behavioral Processes by Class

I. Analysis/Synthesis

Assigns a name or meaning (Ag)

Uses an analogy (An)

Gives a description (Dc)

Identifies a plan or procedure (P)

Poses a general problem (Pg)

Poses a specific problem (Ps)

II. Interpretation

Suggests that more data are needed (Da)

Finds an error and corrects it (E)

Corrects an error by erasing (E)

Observes there is a pattern (Ob)

Qualifies, explains (Qu)

Searches for relevance of data (R)

III. Logical

Reasons by deduction (D)

Makes a generalization (G)

Reasons by induction (I)

Tests the occurrence of a pattern (O)

Summarizes many deductive arguments to reach a final conclusions (T)

Uses trial-and-error to search for a pattern (Ti)

IV. Terminal

Reaches a dead end (De)

Finishes a sequence with completion of thought (.)

Finishes a sequence without completion of thought (/)

Staring, pauses (-)

V. Operational

Adds data (A)

Uses an algorithm, counts (Al)

Mentally counts or writes on paper (Al)

Arranges (Ar)

Makes a comparison (C)

Measures (Me)

States or carries out operation or calculation (Op)

Mentally calculates or calculates on paper (Op)

Uses trial-and-error in specific case (Ts)

VI. Procedural

States is making a table (M)

Writes table or diagram on paper (M)

Manipulates (Ma)

Asks interviewer a question (Q)

Reads (Rd)

Rereads directions (RR)

Interviewer asks student to explain what he is doing (?)

Goes back to problem after finishing others (*)

Makes a statement where coder cannot infer a process (W)

Table 2

Total Percent of Occurrences of 40 Processes by Class and Problem

	Class	Problem				
		ME	NS	PC	PO	TS
I.	Analysis/Synthesis	25.3	27.7	21.8	19.0	15.7
II.	Interpretation	7.5	12.1	8.6	8.7	8.1
III.	Logical	3.0	16.0	15.9	5.1	7.8
IV.	Terminal	19.6	26.2	26.9	20.5	21.7
V.	Operational	33.2	9.1	19.4	24.2	37.3
VI.	Procedural	11.5	8.9	7,4	22.4	9.3

Table 3

Spearman Rank Correlation Coefficients for Comparing

Pairs of Problems for 31 Ordered Pairs and 24 Ordered triples

Problems	Pairs	Triples
ME, NS	0.440*	-0.022
ME, PC	0.322	-0.241
ME, PO	0.427*	0.284
ME, TS	0.131	0.097
NS, PC	0.858**	0.570**
NS, PO	0.742**	-0.124
NS, TS	-0.332	0.150
PC, PO	-0.315	-0.613**
PC, TS	-0.459**	-0.346
PO, TS	0.469**	0.600**

*$p < 0.05$

**$p < 0.01$

Legend. ME-Measurement, NS-Number Systems, PC-Pascal's Triangle, PO-Polyhedra, TS-tessellations

Table 4

Analysis of Variance for Problems and Organization Procedures

Source	SS	df	MS	F-ratio
Procedure	355.938	3	118.646	12.875*
Problem	99.173	4	24.793	10.371*
Procedure x Problem	410.120	12	34.177	7.494*
Subject	306.548	29	10.571	
Procedure x Subject	801.712	87	9.215	
Problem x Subject	277.327	116	2.391	
Procedure x Problem x Subject	1586.981	348	4.560	
Total	3837.799	599		

*p < 0.01

Table 5

Correlations Between Creativity Measures

Measure	F	FL	O	OG	TMC
F	1	0.899*	0.962*	0.848*	0.980*
FL		1	0.860*	0.775*	0.894*
O			1	0.848*	0.990*
OG				1	0.899*
TMC					1

*p < 0.001

Legend: F-Fluency, FL-Flexibility, O-Originality, OG-Organization, TMC-Total Mathematical Creativity

Table 6

Spearman Rank Correlation Coefficients for Comparing High Creativity Groups with other Students for 31 Ordered Pairs and 24 Ordered Triples

Groups	Pairs	Triples
High Fluency	0.395*	-0.142
High Flexibility	0.564**	0.290
High Originality	0.395	-0.142
High Organization	0.572**	0.433*

*p < 0.05

**p < 0.01

Table 7

Correlations Between the Number of Processes Used by Problem and Creativity Measures

Measure	TS	PO	ME	PC	NS	T
F	0.786***	0.451*	0.813***	0.706***	0.582***	0.833***
FL	0.835***	0.444*	0.670***	0.602***	0.580***	0.782***
O	0.801***	0.479**	0.748***	0.667***	0.532**	0.804***
OG	0.723***	0.563**	0.758***	0.805***	0.769***	0.913***
TMC	0.814***	0.502**	0.786***	0.719***	0.607***	0.856***

*p < 0.05

**p < 0.01

***p < 0.001

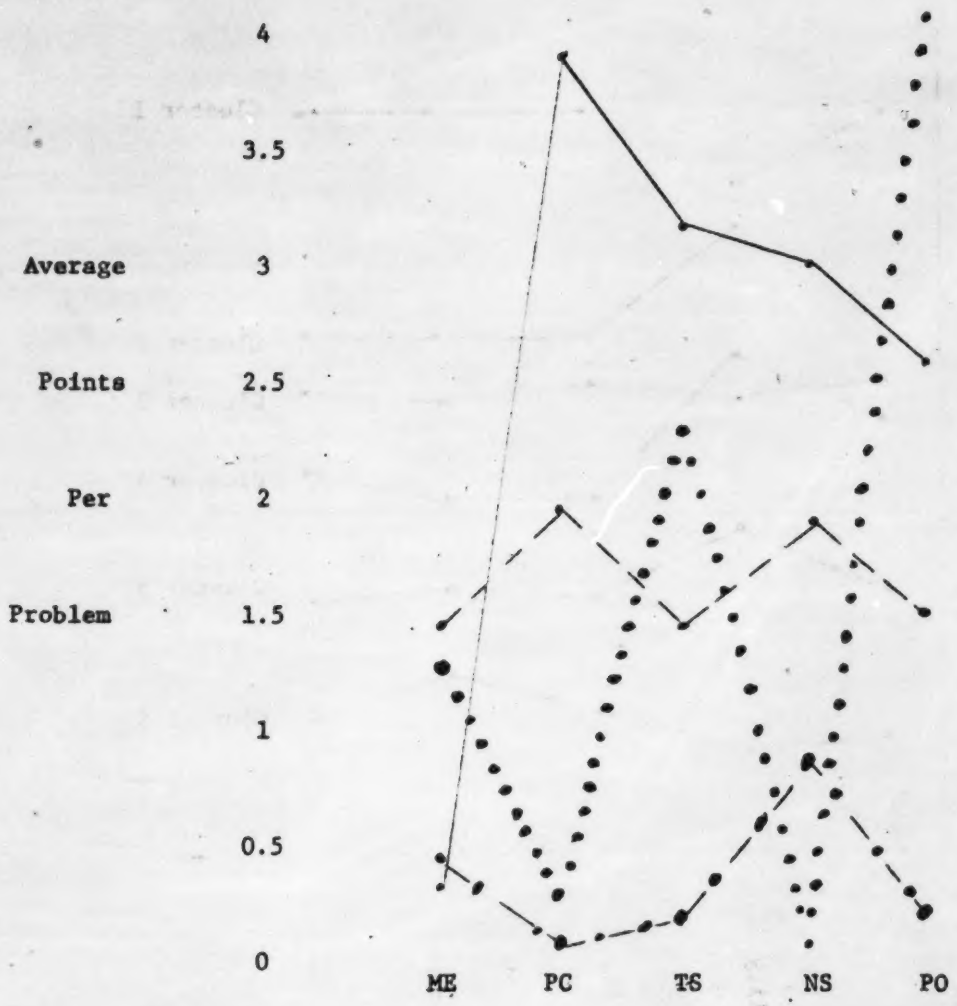
Number System Problem

You have just landed on a strange planet. You are given the following tables using their "number" systems. What guesses can you make about their "numbers"?

*	H	N	T	D	C
H	H	N	T	D	C
N	N	T	D	C	H
T	T	D	C	H	N
D	D	C	H	N	T
C	C	H	N	T	D

#	H	N	T	D	C
H	H	H	H	H	H
N	H	N	T	D	C
T	H	T	C	N	D
D	H	D	N	C	T
C	H	C	D	T	N

Figure 1: A Sample Problem



Posing general problem

Posing specific problem

Covering all combinations in descriptive guesses

Trial-and-error or pattern search to generalization

Figure 2: Mean Number of Organizational Points per Procedure per Problem

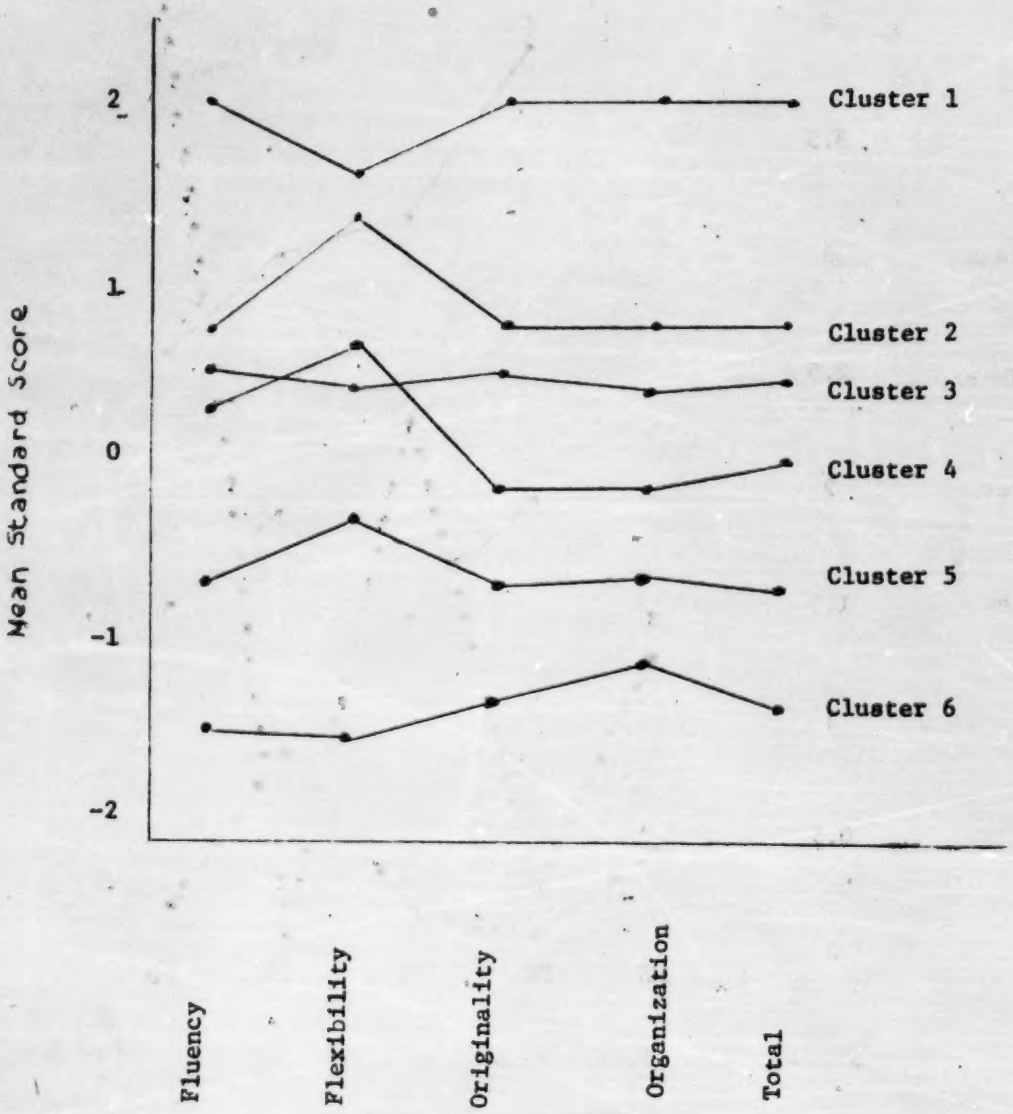


Figure 3: Profiles of Six Clusters for Creativity Measures